1. Consider $f(x) = 2x - 7\ln(x) + e^{2x}$. Using the differentiation rules,

$$f'(x) = 2 - 7\left(\frac{1}{x}\right) + (2)e^{2x} = 2 - \frac{7}{x} + 2e^{2x}.$$

2. Consider $f(x) = 5\ln\left(\frac{1}{x}\right) - e^{-2x} + 2 = 5\ln(x^{-1}) - e^{-2x} + 2 = -5\ln(x) - e^{-2x} + 2$. The rules of differentiation give:

$$f'(x) = -5\left(\frac{1}{x}\right) - (-2)e^{-2x} + 0 = 2e^{-2x} - \frac{5}{x}.$$

3. Consider the function

$$f(x) = \frac{3}{e^{5x}} + 4\ln\left(\frac{1}{\sqrt{x}}\right) - \frac{1}{x} = 3e^{-5x} + 4\ln\left(x^{-\frac{1}{2}}\right) - x^{-1} = 3e^{-5x} - 2\ln(x) - x^{-1}.$$

The rules of differentiation give:

$$f'(x) = 3(-5)e^{-5x} - 2\left(\frac{1}{x}\right) - (-1)x^{-2} = \frac{1}{x^2} - \frac{15}{e^{5x}} - \frac{2}{x}.$$

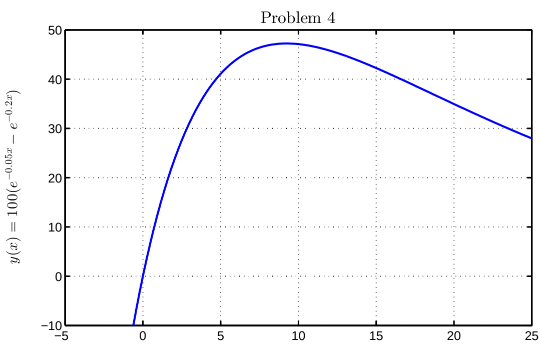
4. Consider the function $y = 100(e^{-0.05x} - e^{-0.2x})$. We take the derivative

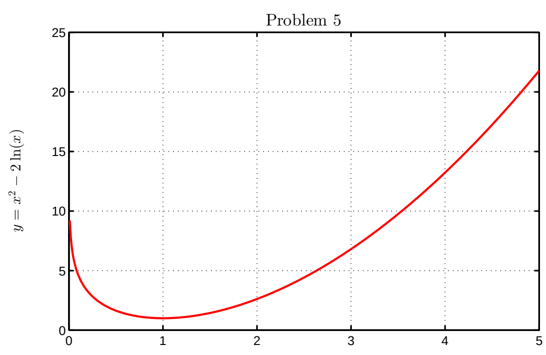
$$y'(x) = 100\left(-0.05e^{-0.05x} - (-0.2)e^{-0.2x}\right) = 5(4e^{-0.2x} - e^{-0.05x}).$$

The domain of $y(x)$ is all x, so there are no vertical asymptotes. The y-intercept is where $x = 0$ or $y = 100(1-1) = 0$. This is also the x-intercept, $(0, 0)$. The horizontal asymptote occurs when $x \to +\infty$, when $y \to 0$. The critical point is where $y'(x) = 5(4e^{-0.2x} - e^{-0.05x}) = 0$, so

$$4e^{-0.2x} = e^{-0.05x} \qquad \text{or} \qquad e^{0.15x} = 4 \qquad \text{or} \qquad x = \frac{\ln(4)}{0.15} \approx 9.2420.$$

Since $x_c = \frac{\ln(4)}{0.15} =\approx 9.2420$, then $y(x_c) \approx 100(e^{-0.05\cdot 9.242} - e^{-0.2\cdot 9.242}) = 47.25$. This is a relative and absolute maximum. The graph is shown below on the left.





5. Consider the function $y(x) = x^2 - 2\ln(x)$. We take the derivative

$$y'(x) = 2x - 2\left(\frac{1}{x}\right) = 2 \quad \frac{x^2 - 1}{x}\Bigg).$$

Domain for y is $0 < x < +\infty$, so there is clearly no y-intercept. There is a vertical asymptote at $x = 0$ (at the edge of the domain), but no horizontal asymptote. This equation cannot be solved for $y = 0$, so x-intercept must be examined in another way. We will establish that the minimum is positive, so no intercepts of the x-axis exist. Critical points occur when $y'(x) = 0$ or $x^2 - 1 = 0$. Since $x > 0$, then $x_c = 1$. $y(1) = 1$, so there is a minimum at (1,1). Besides being a relative minimum, it is also an absolute minimum. The graph is above to the right.